A GENERALIZATION OF ARTIN'S PRIMITIVE ROOT CONJECTURE AMONG ALMOST PRIMES

Paul Péringuey (IECL - Université de Lorraine)

Abstract: Artin's conjecture states that the set of primes for which an integer a different from -1 or a perfect square is a primitive root admits an asymptotic density among all primes. In 1967 C. Hooley [1] proved this conjecture under the Generalized Riemann Hypothesis.

The notion of primitive root can be extended modulo any integer by considering the elements of the multiplicative group generating subgroups of maximal size. One can then look for which elements of a set of integers a given integer is a generalized primitive root, as did S. Li and C. Pomerance for all the integers [2]. I will discuss the set of almost primes for which an integer a is a generalized primitive root, and present results similar to Artin's conjecture for primitive roots.

[1] C. Hooley, *On Artin's conjecture*, J. Reine Angew. Math. **225**, 209–220 (1967; Zbl 0221.10048)

[2] S. Li and C. Pomerance, *On generalizing Artin's conjecture on primitive roots to composite moduli*, J. Reine Angew. Math. **556**, 205–224 (2003; Zbl 1022.11049)

FRACTIONAL PARTS OF BINARY FORMS

Kiseok Yeon (Purdue University)

Abstract: We obtain bounds on fractional parts of binary forms of the shape

$$\Psi(x,y) = \alpha_k x^k + \alpha_l x^l y^{k-l} + \alpha_{l-1} x^{l-1} y^{k-l+1} + \cdots + \alpha_0 y^k$$

with $\alpha_k, \alpha_l, \ldots, \alpha_0 \in \mathbb{R}$ and $l \leq k-2$. By exploiting a variant of Weyl's inequality and inductive arguments, we derive estimates superior to those obtained hitherto for the best exponent σ, depending on k and l, such that

$$\min_{\substack{0 \leq x,y \leq X \\ (x,y) \neq (0,0)}} \|\Psi(x,y)\| \leq X^{-\sigma+\epsilon}.$$

NUMBER OF ELEMENTS OF SMALL NORM IN THE SIMPLEST CUBIC FIELDS

Mikuláš Zindulka (Charles University)

Abstract: We work in a family of the **simplest cubic fields** K_a parametrized by one integer parameter a. Our goal is to estimate the

number of integral elements (up to conjugation and multiplication by units) whose norm is below a certain bound X. We provide an asymptotic estimate depending on a and X up to a multiplicative constant which is independent of the two parameters.

The study of elements of small norm was initiated in a paper by Lemmermeyer and Pethö [1], who showed that the norm of any non-unit integral element of K_a is at least $2a+3$ and that the minimum is attained. The result was further extended by Kala and Tinková [2], who gave an estimate for the number of elements with norm $\leq a^2$. The count is much larger than predicted by the **class number formula**.

We give a natural explanation of this discrepancy and show that if $X \geq a^4$, then the number of elements agrees with the heuristics coming from the class number formula.

[1] F. Lemmermeyer, A. Pethö, *Simplest Cubic Fields*, Manuscripta Math. **88** (1995), 53–58.

[2] V. Kala, M. Tinková, *Universal Quadratic Forms, Small Norms, and Traces in Families of Number Fields*, Int. Math. Res. (2022).

A GENERALIZATION OF THE GRUNWALD-WANG THEOREM AND APPLICATIONS TO RAMSEY THEORY

SOHAIL FARHANGI (Ohio State University)

ABSTRACT: Informally speaking, the Grunwald-Wang theorem shows that for any $n \in \mathbb{N}$, if $x \in \mathbb{Z}$ is locally an n^{th} power, then x is globally an n^{th} power, unless $x = 16$ and $n = 8$. We will discuss a generalization of this theorem to the situation in which we have $x, y, z \in \mathbb{Z}$ such that locally at least one of x, y, and z is an n^{th} power. We will then discuss an application to Ramsey theory and the partition regularity of the equation $ax + by = cw^m z^n$.

SIGNS BEHAVIOUR OF SUMS OF WEIGHTED NUMBER OF COMPOSITIONS

FILIP GAWRON (Jagiellonian University)

ABSTRACT: Let A be a subset of positive integers. For a given positive integer n and $0 \leq i \leq n$ let $c_A(i,n)$ denotes the number of A-compositions of n with exactly i parts. In my talk I will describe our results concerning the sign behaviour of the sequence $(S_{A,k}(n))_{n\in\mathbb{N}}$, where $S_{A,k}(n) = \sum_{i=0}^{n}(-1)^k i^k c_A(i,n)$ is a sum of weighted number number of compositions. Particularly I will describe a broad class of subsets A, such

that the number $(-1)^n S_{A,k}(n)$ is non-negative for all sufficiently large n. I will also mention some examples $A \subset \mathbb{N}_+$ such that the sign behaviour of $S_{A,k}(n)$ is not periodic. The talk is based on a joint work with Maciej Ulas.

THE LEAST COMMON MULTIPLE OF POLYNOMIAL SEQUENCES

Noam Kimmel (Tel-Aviv University)

Abstract: The prime number theorem can be stated as saying that the logarithm of the least common multiple (LCM) of the first N integers is asymptotically equal to N, as was known to Chebyshev. Motivated by this formulation, we look at a generalization – the least common multiple of polynomial sequences. The case of a polynomial in one variable was first studied by Cilleruelo in 2011, who determined the asymptotics of the quadratic case, and has since been explored by various other researchers.

In this talk we consider polynomials in two variables. We discover that already in the quadratic case, there is a range of asymptotic behaviours. We show that for "generic" quadratic polynomials, the growth of log LCM of the values of $F(x, y)$ up to N has order of magnitude $N \log\log N/(\log N)^{1/2}$, but for certain degenerate cases such as $(x + y)^2$ or $x^2 + y^2$, the answers are different.

NUMBER SYSTEMS IN LATTICES

Jakub Krásenský (Charles University)

Abstract: Positional representation of x is $x = \sum_{i=0}^{N} \beta^i a_i$, where the *radix* β is a fixed integer, the *digits* a_i belong to a fixed *alphabet* $\mathcal{A} \subset \mathbb{Z}$ and N is nonnegative. The pair $(\beta, \mathcal{A})$ is called a *number system* or *GNS* if every integer has a *unique* representation. The notion generalises to any ring and is well-studied for orders in number fields. Works of A. Vince, A. Kovács and others show that often it is more natural to replace the ring by a lattice (w.l.o.g. $\mathbb{Z}^d$) and β by a linear operator. In this context, we show an almost complete answer to the question "How many GNSs are there for a given radix?", first studied by D. Matula, and some variants thereof. Some results are joint work with A. Kovács.