ON THE FAREY FRACTION SPIN CHAIN

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Abstract: In 1999, Kleban and Özlük introduced a "Farey fraction spin chain" and made a conjecture regarding its asymptotic number of states with given energy, the latter being given (up to some normalisation) by the number $\Phi(N)$ of 2×2 matrices, arising as products of $\left(\begin{smallmatrix}1 & 0\\ 1 & 1\end{smallmatrix}\right)$ and $\left(\begin{smallmatrix}1 & 1\\ 0 & 1\end{smallmatrix}\right)$, whose trace equals N. Although their conjecture was disproved by Peter ()2001), quite precise results are known on average by works of Kallies–Özlük–Peter–Snyder (2001), Boca (2007) and Ustinov (2013). Time permitting, we shall attempt to convey some of the ideas underlying the above works.

Precise asymptotics for $\Phi(N)$ had hitherto only been known conditionally on the availability of zero-free strips for certain Dirichlet L-functions. However, in this talk we shall see that the question regarding asymptotics for $\Phi(N)$ can be reduced to a problem studied (and solved!) much earlier by Hooley (1958) in a special case and, quite recently, in full generality by Bykovskiĭ and Ustinov (2019). The reduction itself is short and completely elementary.

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