MONOCHROMATIC ARITHMETIC PROGRESSIONS IN BINARY WORDS ASSOCIATED WITH PATTERN SEQUENCES

Bartosz Sobolewski (Jagiellonian University in Cracow)

Abstract: Let $e_P(n)$ denote the number of occurrences of a pattern P in the binary expansion of $n \in \mathbb{N}$. In the talk we consider monochromatic arithmetic progressions in the class of words $(e_P(n) \bmod 2)_{n\geq 0}$ over $\{0,1\}$, which includes the Thue–Morse word $\mathbf{t}$ (for $P = 1$) and a variant of the Rudin–Shapiro word $\mathbf{r}$ (for $P = 11$). So far, the problem of exhibiting long progressions and finding an upper bound on their length has mostly been studied for $\mathbf{t}$ and certain generalizations [1, 2, 3]. The main goal of the talk is to show analogous results for $\mathbf{r}$ and some weaker results for a general pattern P. In particular, we prove that a monochromatic arithmetic progression in $\mathbf{r}$ of difference $d \geq 3$ starting at 0 has length at most $(d+3)/2$, with equality infinitely often. We also compute the maximal length of monochromatic progressions of differences $2^k - 1$ and $2^k + 1$.

[1] I. Aedo, U. Grimm, Y. Nagai, P. Staynova, *On long arithmetic progressions in binary Morse-like words*, preprint, `https://arxiv.org/abs/2101.02056` (2021), 23 pp.

[2] J. F. Morgenbesser, J. Shallit, T. Stoll, *Thue–Morse at multiples of an integer*, J. Number Theory **131** (2011), no. 8, 1498–1512.

[3] O. G. Parshina, *On arithmetic index in the generalized Thue–Morse word*, in: S. Brlek, F. Dolce, C. Reutenauer, É. Vandomme (eds.), Combinatorics on Words, Springer, Cham, 2017, 121–131

ELAZ 2022
Adam Mickiewicz University, Poznań, Poland
August 22–26, 2022