p-ADIC DENSENESS OF FRACTIONS OF VALUES OF POLYNOMIALS WITH INTEGER COEFFICIENTS

PIOTR MISKA (Jagiellonian University in Cracow)

ABSTRACT: For any set $A \subset \mathbb{Z}$ we define its ratio set as

$$R(A) = \{a/b : a, b \in A, b \neq 0\}.$$

The topic of denseness of sets $R(A)$ in positive real half-line generated a lot of literature. Recently, the subject of denseness of these sets in p-adic fieds is also under investigation and, as a new area of study, has many open problems. The talk is devoted to one of them.

Let $f \in \mathbb{Z}[X]$ be fixed. Then, one can ask how large is the set

$$\mathbb{P}_f = \{p \in \mathbb{P} : R(f(\mathbb{Z})) \text{ is dense in } \mathbb{Q}_p\}.$$

During the talk we will show that if f is irreducible over $\mathbb{Z}$, then the set $\mathbb{P}_f$ has the asymptotic density $d_{\mathbb{P}}(\mathbb{P}_f)$ with respect to the set of prime numbers and this density is positive. Moreover, we will focus on the set of al the possible values of $d_{\mathbb{P}}(\mathbb{P}_f)$, when $f \in \mathbb{Z}[X]$ runs over all the irreducible polynomials over $\mathbb{Z}$.