

EXPANSION, DIVISIBILITY AND PARITY

HARALD ANDRÉS HELFGOTT (University of Göttingen)

ABSTRACT: We will discuss a graph that encodes the divisibility properties of integers by primes. We prove that this graph has a strong local expander property almost everywhere. We then obtain several consequences in number theory, beyond the traditional parity barrier, by combining our result with Matomaki-Radziwiłł. For instance: for λ the Liouville function (that is, the completely multiplicative function with $\lambda(p) = -1$ for every prime),

$$(1/\log x) \sum_{n \leq x} \lambda(n)\lambda(n+1)/n = O\left((\log \log x)^{-1/2}\right)$$

which is stronger than well-known results by Tao and Tao-Teräväinen. We also manage to prove, for example, that $\lambda(n+1)$ averages to 0 at almost all scales when n restricted to have a specific number of prime divisors $\Omega(n) = k$, for any “popular” value of k (that is, $k = \log \log N + O(\sqrt{\log \log N})$ for $n \geq N$). This is a joint work with M. Radziwiłł.

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