UNIVERSALITY WITH SCALING

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ABSTRACT: The Voronin universality theorem (for the Riemann zeta-function) shows that any zero-free continuous function $f(s)$ on some compact set $K \subset \{s : 1/2 < \mathrm{Re}(s) < 1\}$ with connected complement, which is analytic in K° may be uniformly approximated to any desired degree of accuracy by $\zeta(s+it)$ for some real t. We consider corresponding approximation theorems when in addition to shifting the argument we allow scaling of the argument and shifting of its value so that $f(s)$ is approximated by $\zeta(\sigma + it + \delta s) + C$ for some constant C. When $\sigma = 1$ this allows us to prove universality in the half plane of absolute convergence which is not possible in the classical case. Also the general method allows us to prove a universality theorem for the Hurwitz zeta-function that is valid also for algebraic irrational parameters, a problem which is wide open in the classical setting. We also briefly discuss some work in progress where corresponding results are proved for $\sigma = \frac{1}{2}$.